Physics III
ISI B.Math
Final Exam : November 26, 2012

Total Marks: 100
Answer any five questions.

1. Marks (5 + 10 + 5)
(a) Suppose an electric field $\mathbf{E}(\mathbf{x}, \mathbf{y}, \mathbf{z})$ has the form $E_x = ax, E_y = 0, E_z = 0$ where a is a constant. What is the charge density? How do you account for the fact that the field points in a particular direction, when the charge density is uniform ?

(b) An infinite plane slab, of thickness $2d$, carries a uniform volume charge density ρ(see figure 4). Find the electric field, as a function of y, where $y = 0$. Plot E vs y, calling E positive when it points in the $+y$ direction and negative when it points in the $-y$ direction.

(c) If it was discovered that the electrostatic force between two charges was proportional to $\frac{1}{r^3}$ rather than $\frac{1}{r^2}$, would it still be possible to associate a scalar potential function with such a force? Will Gauss's law continue to hold ? Explain.

2. Marks (12 + 8)
(a) A uniform line charge λ per unit length is placed on an infinite straight wire, a distance d above a grounded conducting plane. (Let's say the wire runs parallel to the x-axis and directly above it and the conducting plane is the $x - y$ plane.) Find the potential in the region above the plane.

(b) Find the charge density σ induced on the conducting plane.

3. Marks (9 + 5 + 3 + 3)
(a) Three long straight parallel wires are located as shown in figure 2. One wire carries current $2I$ into the paper, each of the others carries current I in the opposite direction. What is the strength of the magnetic field at the point P_1 and at the point P_2 ?

(b) Give the magnitude and direction of the Poynting vector $\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E} \times \mathbf{B})$ at the surface of a long straight wire of circular cross-section carrying a current I. The radius of the wire is b, and the resistance per unit length is R.

(c) The claim is that the the energy stored in a magnetic field B is given by $\int B^2 d\tau$ where the integral is over all space. (To think of this in more concrete terms, you can think of the energy stored in the inductance in an LC circuit). Justify how energy can be stored in a magnetic field though the force associated with a magnetic field can do no work.

(d) A particle of mass m and charge q is moving under the influence of an electric field $\mathbf{E}(\mathbf{x}, \mathbf{t})$ and a magnetic field $\mathbf{B}(\mathbf{x}, \mathbf{t})$. Is it necessarily true that the mechanical energy of the particle is conserved ? If not, explain why this does not lead to a breakdown of the general principle of conservation of energy.
(for part (c) and (d), only qualitative reasoning will suffice).

4. Marks (4 + 5 + 5 + 6)

A metal crossbar of mass m slides without friction on two long parallel conducting rails a distance b apart as shown in fig. 1. A resistor R is connected across the rails at one end ; compared with R the resistance of the bar and rails is negligible. There is a uniform magnetic field $\mathbf{B}$ perpendicular to the plane of the figure. At time $t = 0$, the crossbar is given a velocity v_0 toward the right.

(a) If the bar moves to the right at speed v, what is the current in the resistor? In which direction does it flow?

(b) Find the speed v of the crossbar at time $t > 0$.

(c) How far does the crossbar go before it stops ?

(d) Show that the motion of the crossbar is consistent with the conservation of energy by showing that the energy dissipated in the resistor is exactly equal to the kinetic energy lost by the crossbar.

5. Marks (12 + 8)

(a)A short solenoid (with length l and radius a with n_1 turns per unit length) lies on the axis of a very long solenoid(radius b,n_2 turns per unit length) as shown in figure 3. Current I flows in the short solenoid. What is the flux through the long solenoid? Find the mutual inductance.

(b) Maxwell's equations admit plane monochromatic wave solutions propagating in the z direction of the form $\mathbf{E}(z,t) = \mathbf{E_0}e^{i(kz-\omega t)}, \mathbf{B}(z,t) = \mathbf{B_0}e^{i(kz-\omega t)}$,where $\mathbf{E_0}, \mathbf{B_0}$ are the complex amplitudes of the electric and magnetic fields respectively and are constant. k and ω are the wave number and angular frequency of the waves respectively. Use Maxwell's equations to show that these waves are transverse.

6. Marks (4 + 5 + 5 + 6)

(a) Write down the full set of Maxwell's equations in differential form with sources ρ and $\mathbf{J}$.

(b) Obtain the continuity equation : $\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0$ from Maxwell's equations, where the symbols have their usual meanings. Show that the above equation represents the conservation of charge.

(c) Show that a magnetic field $\mathbf{B}$ and electric field $\mathbf{E}$ that is a solution to Maxwell's equations can always be written as

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\mathbf{E} = -\nabla \phi - \frac{\partial \mathbf{A}}{\partial t}$$

where ϕ is a scalar function and $\mathbf{A}$ is a vector field.

(d) Show that , for Maxwell's equations in vacuum, each Cartesian component of $\mathbf{E}$ and $\mathbf{B}$ satisfies the 3-D wave equation

$$\nabla^2 f = \frac{1}{c^2} \frac{\partial^2 f}{\partial t^2}$$

with $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$. (Note that :$\nabla \times \nabla \times \mathbf{A} = -\nabla^2 \mathbf{A} + \nabla(\nabla \cdot \mathbf{A})$)

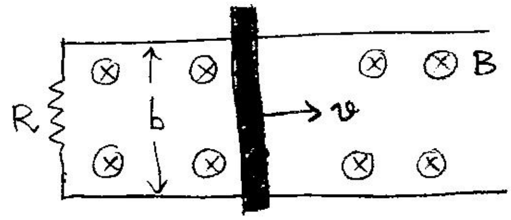

Figure 1

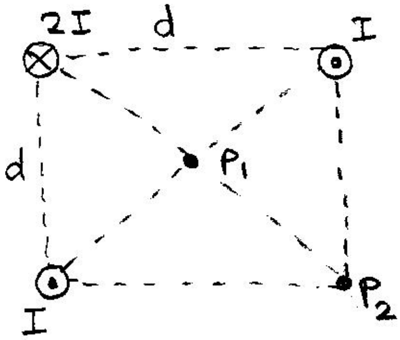

Figure 2

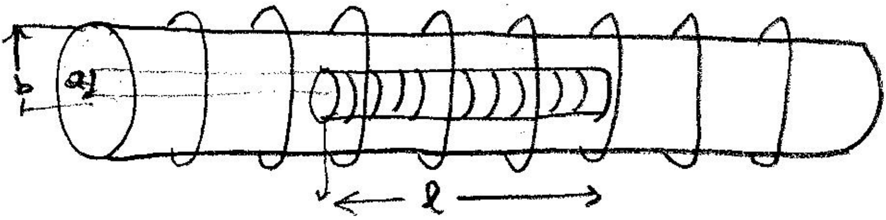

Figure 3

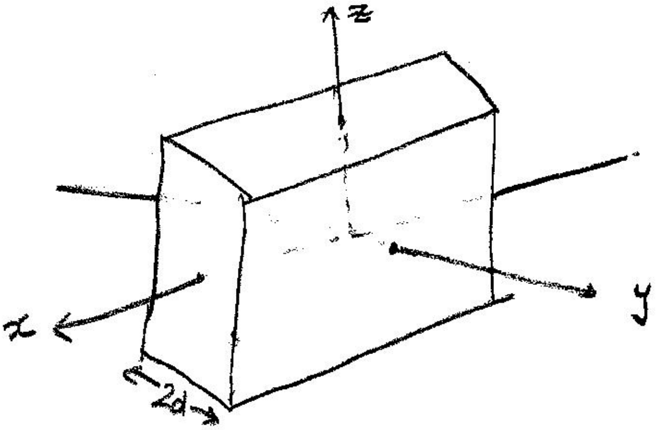

Figure 4